Covariance Matrix Jumps in High-Frequency Financial Markets

Yuri Hupka[†] and Louis R. Piccotti[‡]

October 2021

Abstract

Jumps and cojumps are examined in the covariance matrices of high-frequency financial markets. We propose a new method for identifying intraday volatility jumps in the diffusive covariance matrix of asset pairs. Our method avoids model misspecification errors, is able to identify multiple intraday jumps, and provides standard normally distributed test statistics. Performance is tested using Monte-Carlo simulated and tick level foreign exchange market data. Empirical evidence reveals the presence of jumps in both price and covariance matrices, with covariance jumps occurring at an increased rate. Simulation results demonstrate our proposed method correctly identifies covariance jumps at a statistically high rate and with no spurious detection.

Keywords: Jumps; Cojumps; Foreign Exchange Market

JEL Codes: G12, C58, F31

[†]Corresponding Author. Department of Finance, Spears School of Business, Oklahoma State University, Stillwater, OK 74078, e-mail: hupka@okstate.edu

[‡]Department of Finance, Spears School of Business, Oklahoma State University, Stillwater, OK 74078, e-mail: louis.r.piccotti@okstate.edu, tel: +1-405-744-8666.

The data that support the findings of this study are available from the corresponding author upon reasonable request. All errors are the authors sole responsibility.

1. Introduction

Previous theoretical research has shown that jumps in asset prices, as well as jumps in volatility, have important implications for the asset allocation problem (Branger et al., 2008; Liu et al., 2003; Liu and Pan, 2003). However, an empirical difficulty in testing these models lies in disentangling the identification of jumps in the asset prices from jumps in the diffusion covariance matrix. Considerable advances have been made towards asset price jump identification, such as those by Barndorff-Nielsen and Shephard (2004), Barndorff-Nielsen and Shephard (2006), and Andersen et al. (2007).

We advance the literature by examining jumps in the diffusive covariance matrix at the intraday frequency. By assuming that the diffusion covariance matrix evolves at a lower frequency than the financial asset price series, we extend the jump identification of Barndorff-Nielsen and Shephard (2006) and Andersen et al. (2007) into a two-step methodology. In the first step, price jumps are identified using strategies similar to the prior literature. In the second step, we use the filtered diffusive covariance matrix attained from the first step to identify jumps covariance jumps in asset pairs. Therefore, our covariance jump methodology also falls within the literature proposing multi-frequency estimators (Zhang, 2006; Zhang et al., 2005).

An extant literature has examined the continuity properties of volatility by searching for jumps both in the CBOE Volatility Index (VIX) as well as in individual securities returns[1]. The drawback of only identifying jumps in the VIX is that the volatility jump dynamics cannot be generalized to identifying jumps in individual assets without additionally specifying a factor model for returns, which is subordinated on the aggregate VIX index. Latter methods identifying volatility jumps directly in the underlying traded securities resolve the

[1] Jump studies using the VIX include Todorov (2015), Todorov and Tauchen (2011), and Todorov et al. (2014). For securities returns studies see Bibinger et al. (2017), Caporin et al. (2015), and Jacod and Todorov (2010).

subordinating specification issue. The point-change method of Bibinger et al. (2017) also has the beneficial property of being able to identify multiple intraday jumps.

Our proposed model for identifying volatility jumps advances this literature in several ways. First, the test statistics only require observed price data thus avoiding model specification errors. Second, since it is a two-step application of the Barndorff-Nielsen and Shephard (2006) and Andersen et al. (2007) jump identification methods, we are able to identify multiple volatility jumps at an intraday frequency. Third, the test statistics are standard normally distributed.[2] As a result, tests for asset volatility jumps and cojumps, both within and across asset classes, are simply the sum of the test statistics and follow a chi square distribution.

The quality of our test is examined using a Monte-Carlo simulation study. Using simulated covariance paths under stochastic volatility and affine jump process, we construct price pairs resulting in the affine double jump processes described in Equations (1) and (2).[3] Our test reliably detects jumps in high frequency data with minimal probability for incorrect spurious detections. Under an equivalent volatility calibration and the commonly used 5 *minute* observation block setting, we detect jumps at a rate of 99.87%. Alternatively, decreasing observational frequency leads to a 91.8% detection rate. The spurious detection rates across all calibrations is shown to statistically be zero. Various calibrations demonstrate reliable test performance under a wide variety of jump sizes and observational frequencies analagous to practical use. Though a caveat should be noted that high test quality holds provided a reasonably large mean jump magnitude and tight observational frequency is concurrently utilized.

Lastly, we empirically test our proposed jump identification model the currency pairs derived from 15 individual FOREX series. A broad suite of tick level foreign exchange rate prices were collected from Gain Capital over the period of January to December 2017. We

[2]This feature is also present in Andersen et al. (2007).

[3]The covariance processes restricted to be positive semi-definite. Visual representation of simlated covariance and pricing pairs can be found in Figure 1.

find that jumps occur in the price, variance, and pair covariances at mean rates of 1.06%, 1.43%, and 1.53%, respectively, with negative jumps occurring at slightly lower frequencies to their positive counterparts. Diffusive covariance jumps occur in 24.76% of days, suggesting they occur more often than price jumps albeit over fewer days.

Our empirical results have broad implication to research involving second momemt dynamics. Examples include covariance risk (Harvey, 1991), investment return premia (Moskowitz, 2003), and the ICAPM model (Guo and Whitelaw, 2006). Such works often take an ad-hoc approach to covariance process specification, an issue is especially omnipresent in GARCH forecasts related to hedging (Bali, 2008), derivatives and options (Duan et al., 2006), and FOREX pricing (Fiszeder et al., 2019). The volatility forecast literature has recently extended the second moment forecasting to include GARCH-Jump features (Lin et al., 2013; Callot et al., 2017), the underlying specification of the jump intensity and magnitude parameters is often opaque. By utilizing our test prior to process estimation, future research would more closely parameterize models according to their endogenous sample features.

The remainder of the paper is organized as follows. Related literature is briefly discussed in Section 2. In Section 3 we present the model for covariance matrix jump identification. Empirical results from our simulation and data studies are reported in Section 4. Section 5 concludes.

2. Literature Review

Efforts to specify the nature of jump processes have provided a plethora of approaches. Early works often utilize jump diffusion models while employing a number of variants. Such methodological variations include using GMM estimators (Chernov and Ghysels, 2000; Pan, 2002), simulated method of moments (Duffie and Singleton, 1993), efficient method of moments (Gallant and Tauchen, 1996; Andersen et al., 2002), and indirect inference methods (Gourieroux et al., 1993).

Heston (1993), Melino and Turnbull (1995), and Bates (1996) examine combined jump diffusion models, advocating for the inclusion of jumps to the volatility process. Duffie et al. (2000) generalized their approach to include affine double jumps, simulataneous jumps in both volatility and prices. They argue that models which include volatility jumps perform significantly better. A number of authors have found emprical support for their claim (Eraker et al., 2003; Eraker, 2004; Liu and Pan, 2003; Bates, 2000; Wu, 2003). Studies using Monte-Carlo Markov Chain simulated data have gnerated additional empirical support (Jacquier et al., 2002; Johannes, 2004).

Todorov and Tauchen (2011), Todorov et al. (2014), and Todorov (2015) have recently studied jump activity in the VIX. These works employed nonparametric statistics to test for asymmetric volatility jumps. All conclude the VIX exhibits a pure jump semi-martingale process with no continuous component. Todorov et al. (2014) further evaluate their parameter estimates using high frequency VIX data and S&P index returns, generating strong supportive evidence for their conclusions.

Beyond the theoretical, covariance is of primary empirical importance for portfolio theory and asset selection. It is well documented that investment strategies which account for dynamic covariance structures are more efficient than those that ignore second moments (Moskowitz, 2003). In recent decades, a wider literature has explored the many implications of second moments. Particular focus has been on characterizing the risk return relationship. Harvey (1991) explores the reward per unit of second moment risk for a global asset portfolio. He finds that time-varying covariance capture some, but not all, of cross-country dynamic return behavior. The empirical strategy employed exploits time variation in country specific expected return and conditional covariance with the world portfolio return. In addition to global factors, return predictions are formed on 'local instruments', of which foreign exchange rate changes are included. While the main goal is calculating a 'world price' of covariance risk, the underlying covariance structure assumes a simplified stochastic structure. Incorrect specification of this structure will likely lead to an inefficient global portfolio, an issue which the author notes.

Moskowitz (2003) identify a broad association between investment strategy return premia and conditional second moments. They utilize a standard Geometric Brownian Motion GARCH model to characterize time-varying covariance volatility. The authors argue that time variation in return second moments curtails the ability of time-invariant factors to properly capture return volatility dynamics. Improperly captured dynamics are found to lead to inefficient investment and sub-optimal out–of-sample mean value portfolios.

Covariance risk premia has been further explored under an ICAPM setting. Guo and Whitelaw (2006) estimate covariance models using decomposed risk and hedge components. Bali (2008) extends the multivariate GARCH setting to include estimates of conditional covariances. Such estimates are then used to form portfolios on firm and industry variables. Bali and Engle (2010) later extend this model to include dynamic variation in conditional correlation. Rossi and Timmermann (2015) relax the assumption of linearity in the covariance process, finding improved performance when utilizing nonparametric projections of various state variable sets.

These works not only illustrate the importance of properly characterizing covariance processes, but also have led to a variety of suggested methods for including jump components within GARCH models. Their empirical use often applied to options and derivatives pricing. Although a significant improvement on previous models, these works have a key shortcoming of parameterizing the covariance jump processes in an ad-hoc fashion. Specifically, this relates to the calibration of jump intensity (λ) and magnitude (σ). Duan et al. (2006) consider limiting models of diffusive prices, correlated jumps, and volatilities within the GARCH-Jump processes. However, instead of defining exact calibrations they employ a range of possible parameter values. Similarly, Callot et al. (2017) uses a combination of one-step-ahead forecasts and exponentially weighted moving averages (EWMAs), with a high smoothing parameter, for calibration. Lin et al. (2013) incorporates a jump process to develop a lattice model for options pricing, finding jump effects in both in- and out-of-the money options. Again, they rely on posited distributions to match future period intensity and size probabilities.

Empirical application of GARCH modeling to FOREX individual currency pairs can be seen in the BEKK model results of Fiszeder (2018). Fiszeder et al. (2019) extends their prior work to range-based and high/low price DCC models. Both works use single FOREX closing price series for GARCH optimization. Covariance jump which require only price series, and therefore no additional components or state variables, to identify jump occurrences would be of particular value in such cases.

Omission or mischaracterization of a jump component would fall under many potential misspecification umbrellas of the prior literature. While we do not explore each implication individually, we broadly inform the literature in two ways. First, we provide evidence that jumps occur in the covariance of asset pairs, along with their probability and size at multiple frequencies. Particularly, our empirical results directly relate to jumps in foreign currency pairs. Second, our proposed test provides future researchers a straightforward and way to include and parameterize covariance jump components in future asset pricing models. Additionally, these methods are generalizable across other assets or commodities for which a pricing series is obtained. These developments are further illustrated via Monte-Carlo simulation.

3. Identification Model

We assume that asset prices are observed on the unit interval $[0, 1]$ on a fixed time grid, where the time between observations is denoted by Δ. Therefore, there are $T = \Delta^{-1}$ observations and we are concerned with the limiting case where $\Delta \to 0$. For now we assume that all prices are observed synchronously. This abstracts from the Epps (1979) effect where realized correlations are biased towards zero (due to non-synchronous trade) when higher-frequency data is used.[4]

Let the vector of M log price changes be described by the following doubly stochastic process of Duffie et al. (2000):

$$dP_t = \alpha_t^P dt + \Theta_t^P dW_t^P + \Xi_t^P dN_t^P \tag{1}$$

$$dVech(\Sigma_t) = \alpha_t^\Sigma dt + \Theta_t^\Sigma dW_t^\Sigma + \Xi_t^\Sigma dN_t^\Sigma \tag{2}$$

where $P_t = (P_{t,1}, P_{t,2}, \ldots, P_{t,M})'$ is the vector of log asset prices. $\alpha_t = \alpha_{t,1}, \alpha_{t,2}, \ldots, \alpha_{t,M})'(\alpha_t^\Theta)$ is the vector of per annum log price (covariance) drift coefficients. $\Theta_t(\Theta_t^\Sigma)$ is the $(M \times M)$ asset (covariance) log return covolatility matrix such that the covariance matrix is $\Sigma_t = \Theta_t \Theta_t'$.[5] $W_t = (W_{t,1}, W_{t,2}, \ldots, W_{t,M})'(W_t^\Sigma)$ is the vector of independent standard Brownian motions. $\Xi_t(\Xi_t^\Sigma)$ is the $(M \times M)$ covolatility matrix of finite activity jumps in log prices (covariances). $N_t = (N_{t,1}, N_{t,2}, \ldots, N_{t,M})'(N_t^\Sigma)$ is the vector of counts of the number of jump events that have arrived for each asset (covariance) up to time t. $Vech(\cdot)$ denotes the $\left(\frac{M(M+1)}{2} \times 1\right)$ vector, which stacks the elements on and below the main diagonal of a square matrix, moving

[4]See Hayashi and Yoshida (2005) for an analysis of the bias of the realized covariance estimator as the sampling frequency increases with asynchronously traded assets.

[5]$\Sigma_t^\Sigma = \Theta_t^\Sigma \Theta_t'^\Sigma$

from left to right.[6]

Similar to the to the log price processes, the covariance process is composed of a continuous component as well as a discontinuous component. We do not assume that Θ_t^{Σ} or Ξ_t^{Σ} are diagonal, which means that covariances are allowed to have correlated diffusive moves as well as correlated jumps. Further, we do not restrict the matrix given by $(dW_t, dN_t, dW_t^{\Theta}, dN_t^{\Theta})'(dW_t, dN_t, dW_t^{\Theta}, dN_t^{\Theta})$ to be diagonal. This allows for the leverage effect (Black, 1976) as well as for volatility feedback effects (Campbell and Hentschel, 1992; Bekaert and Wu, 2000). Since variances cannot be negative, a stochastic differential equation (SDE) representation such as the Heston (1993) model can be used for the variance terms in Equation (2).

Let K be an integer and $\frac{T}{K}$ be the largest integer that is less than or equal to $\frac{T}{K}$. In practice, K can be the number of observations in one trading day, in one hour, or in one 20-minute period, for example. Divide the sample into K non-overlapping blocks, each containing $\frac{T}{K}$ Δ-period observations. Let t^k be the set of observations in the k'th block of data and let t_l^k denote the l'th observation of the k'th block of data for $k = \{1, 2, \ldots, K\}$.

In order to attain the diffusive covariance matrix of returns Σ_t, the effects of log price jumps need to first be removed. Denote the Δ-period log return for the log price vector as $r_{t_l^k} = P_{t_l^k} - P_{t_l^k - 1}$. Following from the results of Andersen et al. (2001) Barndorff-Nielsen and Shephard (2004), and Barndorff-Nielsen and Shephard (2006), the bipower covariation can be subtracted from the realized covariance to identify the jump covariance of a stochastic process. That is, in the limit as $\Delta \to 0$:

$$RCV(P, t^k) - BCV(P, t^k) \to \sum_{t^k} \Xi^P(t_s^k)\Xi'^P(t_s^k)\Delta N^P(t_s^k)\Delta N'^P(t_s^k), \tag{3}$$

[6]For example, $Vech\left(\begin{bmatrix}\alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3}\\ \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3}\\ \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3}\end{bmatrix}\right) = (\alpha_{1,1}, \alpha_{2,1}, \alpha_{3,1}, \alpha_{2,2}, \alpha_{3,2}, \alpha_{3,3})'$

where:

$$\begin{aligned}
& RCV(P,t^k) = [P]_{t^k} \\
& = \lim_{\Delta \to 0} \sum_{l=1}^{\#(t^k)} r_{t_l^k} r'_{t_l^k} \\
& \to \int_{t^k} \Theta(t_s^k)\Theta'(t_s^k)ds + \sum_{t^k} \Xi^P(t_s^k)\Xi'^P(t_s^k)\Delta N^P(t_s^k)\Delta N'^P(t_s^k)
\end{aligned} \tag{4}$$

is the realized covariance for the k'th block and:

$$\begin{aligned}
& BCV(P,t^k;1) = \{P;1\}_{t^k} \\
& = \lim_{\Delta \to 0} \sum_{l=2}^{\#(t^k)} \Gamma_{t_l^k}||\Gamma_{t_l^k}|' \\
& \to \int_{t^k} \Theta(t_s^k)\Theta'(t_s^k)ds
\end{aligned} \tag{5}$$

is the bipower covariation for the k'th block. $\#(t^k)$ denotes the cardinality of the set t^k (the number of observations in t^k). The associated discrete estimators for $RCV(P,t^k)$ and $BCV(P,t^k;q)$ are:[7]

$$\widehat{RCV}(P,t^k) = \sum_{t^k} \Gamma(t_l^k)\Gamma'(t_l^k), \tag{6}$$

$$\widehat{BCV}(P,t^k;1) = \mu_1^{-2} \sum_{t^k} |\Gamma_{t_l^k}||\Gamma_{t_l^k}|' \tag{7}$$

where $\mu_1 = \sqrt{\frac{2}{\pi}}$. We use Equation (6) to filter the jump component out of the observed log return series by calculating $\widehat{BCV}(P,t^k;1)$ for each k block of data. The time series of unique diffusion covariance elements left are:

$$\int_{t^k} Vech(\Sigma(t_s^k))\, ds = \int_{t^k} \alpha^{\Sigma}(t_s^k)\, ds + \int_{t^k} \Theta^{\Sigma}(t_s^k)\, ds + \Sigma_{t^k} \Xi^{\Sigma}(t_s^k)\Delta N^{\Sigma}\, ds \tag{8}$$

[7]See Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2004), and Barndorff-Nielsen and Shephard (2006) for a thorough presentation of relevant discrete estimators.

which is time varying across each of the K blocks of data. As $\Delta \to 0$, $k \to \infty$, with $\frac{T}{k} \to c$ for some constant c.[8] As data frequency increases, blocks covering a smaller period of calendar (intraday) time can be formed. Within the specified limits, block estimates of the integrated covariance converge to the latent spot covariance of that blocks' intraday time.

We use the recovered $Vech(\Sigma)$ estimates to examine the realized covariance of $Vech(\Sigma)$, $RCV(\Sigma)$, and the bipower covariataion of $Vech(\Sigma)$, $BCV(\Sigma)$. K non-overlapping blocks of data are collected into H non-overlapping blocks, resulting in each new block containing $\frac{K}{H}$ blocks.

Let τ^h be the set of blocks and let t_l^h denote the l'th block, in the h'th broad block of data.[9] $\widehat{RCV}(P, t^k)$ and $\widehat{BCV}(P, t^k; 1)$ are calculated on the K narrow blocks of data and $\widehat{RCV}(\Sigma, \tau^h)$ and $\widehat{BCV}(\Sigma, \tau^h; 1)$ are calculated using the $\frac{K}{H}$ blocks of narrow data within each of the broad H blocks. Therefore, a two-step RCV and BCV estimator is being employed.

By Equation (2), $dVech(\Sigma_t)$ has the same stochastic properties as dP_t. Therefore, the following two convergence results of Equation (9) and Equation (10) hold:

$$\begin{aligned}
&RCV(\Sigma, \tau^h) = [\Sigma]_{\tau^h} \\
&= \lim_{\Delta \to 0} \sum_{l=1}^{\#(\tau^h)} |\Gamma_{t_l^h}||\Gamma_{t_l^h}|' \\
&\to \int_{\tau^h} \Theta^{\Sigma}(t_s^k)\Theta'^{\Sigma}(t_s^k)ds \\
&+\sum_{\tau^h} \Xi^{\Sigma}(t_s^h)\Xi'^{\Sigma}(t_s^h)\Delta N^{\Sigma}(t_s^h)\Delta N'^{\Sigma}(t_s^h)
\end{aligned} \tag{9}$$

$$\begin{aligned}
&BCV(\Sigma, \tau^h; 1) = \{\Sigma; 1\}_{\tau^h} \\
&= \lim_{\Delta \to 0} \sum_{l=2}^{\#(\tau^h)} |\Gamma^{\Sigma}_{t_l^h}||\Gamma^{\Sigma}_{t_{l-1}^h}|' \\
&\to \int_{\tau^h} \Theta^{\Sigma}(t_s^h)\Theta'^{\Sigma}(t_s^h)ds
\end{aligned} \tag{10}$$

where $\Gamma(t_l^h, \Sigma) = Vech(\Sigma_{t_l^h}) - Vech(\Sigma_{t_{l-1}^h})$, and $\#(\tau^h)$ is the number of t^k blocks of data

[8] Correspondingly, $\int_{t^k} Vech(\Sigma(t_s^k))\, ds \to Vech(\Sigma(t))$ where $t \in t^k$.

[9] The h'th block is characterized by $h = 1, 2, \ldots, H$.

within τ^h. We then subtract the sample bipower covariation, from the realized covariation to identify the integrated diffusive covariance matrix for the covariance process:[10].

$$\widehat{RCV}(\Sigma, \tau^h) - \widehat{BCV}(\Sigma, \tau^h; 1) \rightarrow \sum_{\tau^h} \Xi^{\Sigma}(t_s^h)\Xi'^{\Sigma}(t_s^h)\Delta N^{\Sigma}(t_s^h)\Delta N'^{\Sigma}(t_s^h) \tag{11}$$

Therefore, our model has the restriction that it identifies jumps in the covariance matrix at a lower frequency than jumps in the log price series. However, asymptotically as $\Delta \rightarrow 0$, the discrete jump term ($\Xi^{\Sigma}\, dN^{\Sigma}$) dominates the continuous Brownian motion term ($\Theta^{\Sigma}\, dW^{\Sigma}$) which converges to zero with Δ. [11,12] As a result, if t^* is the true covariance jump time, then the identified covariance jump time converges to t^* from the right as $\Delta \rightarrow 0$.

Andersen et al. (2007) and Lee and Mykland (2008) show that intraday jumps in prices can then be identified with the following strategy:

$$\kappa\left(i, t^k\right) = \Gamma\left(i, t_l^k\right) \bullet \left(\frac{|\Gamma\left(i, t_l^k\right)|}{\sqrt{\#(t^k)^{-1} \cdot \widehat{BCV}\left(i, i, t^k; 1\right)}} > \Phi_{1-\frac{\beta}{2}} \right) \tag{12}$$

where $\left(\frac{|\Gamma\left(i,t_l^k\right)|}{\sqrt{\#(t^k)^{-1}\cdot\widehat{BCV}\left(i,i,t^k;1\right)}}\right)$ is distributed standard normally, $\Phi_{(1-\frac{\beta}{2})}$ denotes the critical value from the standard normal distribution, $\Gamma(i, t_l^k)$ denotes the log return on asset i at time t_l^k, and $\widehat{BCV}(i, i, t^k; 1)$ denotes the $[i, i]$'th element of the sample BCV matrix for block t^k. Similar to Equation (12), intraday jumps in the diffusive covariance matrix are identified with the following strategy:

$$\begin{aligned} &\kappa\left(Vech(\Sigma), i, \tau^h\right) = \\ &\quad \Gamma\left(Vech(\Sigma), i, t_l^h\right) \bullet \left(\frac{|\Gamma\left(Vech(\Sigma), i, t_l^h\right)|}{\sqrt{\#(\tau^h)^{-1} \cdot \widehat{BCV}\left(Vech(\Sigma), i, \tau^h; 1\right)}} > \Phi_{1-\frac{\beta}{2}} \right) \end{aligned} \tag{13}$$

[10]Bipower covariation: $\widehat{BCV}(\Sigma, \tau^h; 1)$
Realized covariation: $\widehat{RCV}(\Sigma, \tau^h)$

[11]$\Theta^{\Sigma}\, dW^{\Sigma} = \Theta^{\Sigma} Z^{\Sigma} \sqrt{\Delta}$

[12]Where $Z^{\Sigma} \sim N(0, I_M)$ is an $M \times 1$ vector of standard normally distributed random variables.

where $i \in \left\{1, 2, \ldots, \frac{N(N+1)}{2}\right\}$.

A covariance matrix containts $\frac{N(N+1)}{2}$ unique elements. Additionally, each jump test statistic is standard normally distributed. Therefore, a joint test for a full covariance matrix jump is the sum of the squared jump test statistics. Resultant joint test statistics are χ^2 distributed.

Proposition 1. *The joint test for covariance matrice jumps is given by:*

$$\kappa\left(Vech(\Sigma), t^k\right) = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{\left|\Gamma\left(Vech(\Sigma), i, t_l^h\right)\right|}{\sqrt{\#(\tau^h)^{-1} \cdot \widehat{BCV}\left(Vech(\Sigma), i, \tau^h; 1\right)}}\right)^2 \sim \chi^2 \frac{N(N+1)}{2} \tag{14}$$

Proof. The sum of M squared standard normal random variables is distributed as χ^2_M. Therefore, the sum of the $\frac{N(N+1)}{2}$ squared jump test statistic kernels is distributed $\chi^2_n \frac{(N(N+1)}{2}$.[13] $\square$

In Proposition (1) we show that a covariance matrix jump occurs when there is a statistically significant change in the covariance of at least 2 assets. A drawback of the joint test of Equation (14) is that it cannot identify which assets experience covariance jumps. However, by applying Equation (13) to individual covariance matrix elements, cojumps in individual asset pair covariances are identified.

4. Empirical Results

[13]Jump test statistic kernels: $\left(\frac{\left|\Gamma\left(Vech(\Sigma), i, t_l^h\right)\right|}{\sqrt{\#(\tau^h)^{-1} \cdot \widehat{BCV}\left(Vech(\Sigma), i, \tau^h; 1\right)}}\right)^2$

4.1. FOREX Data Testing

We apply our test on 15 bilateral USD exchange rates using tick-by-tick data obtained from Gain Capital for the sample period January to December, 2017. Currencies included are: Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Franc (CHF), Danish Krone (DKK), Euro (EUR), Hungarian Forint (HUF), Japanese Yen (JPY), Mexican Peso (MXN), New Zealand Dollar (NZD), Russian Ruble (RUB), Swedish Krona (SEK), Singapore Dollar (SGD), Turkish Lira (TRY), and South African Rand (ZAR). We specifically determine the dynamics of the individual currency series price and variance, as well as covariance jump dynamics of the FOREX pairs.

In Table 1 we reported the summary statistics of daily returns for each currency pair exchange rate. Daily prices are formulated at one day sampling frequencies. Daily returns are calculated as log price difference of the daily prices, following $r_{t_l^k} = P_{t_l^k} - P_{t_{l-1}^k}$ where $k = 1$ day. The mean daily return is -0.015%. Standard deviation ranges from 0.243 to 0.874. The correlation between standard deviation and price jump prevalence is 0.66. The averages for skewness and kurtosis are -0.017 and 1.197, respectively.

[INSERT TABLE 1]

Following Equation (12), we test for jump in the prices of individual exchange rates, using a significance level of 5% and 5-minute blocks. As demonstrated in the latter simulation exercise, Although our test is robust to wide array of frequency specifications, jump test power increases at higher frequencies. Therefore, we choose to present our results for subsequent tables with respect to a 5-minute frequency as is common to the literature.

We first test for jumps in the Forex prices series, presenting our results in Table 2. We find that price jumps occur in 133 of the sample days at a mean occurrence rate of 1.06%. RUB (1.39%) and TRY (1.22%) exibit the largest average jump prevalences, while CHF

(0.89%) exitibits the smallest. We also find that positive jumps occur 0.02% more often than negative jumps, while tending to be slightly smaller. Our average daily return jump prevalence is smaller than that of Lee and Wang (2019), who find 1.53% on average for 17 country exchanges over the years 1999-2015. Overall, our jump values are comparable to theirs. Minor discrepancies are likely due to their inclusion of high-prevalence outliers such as India (5.2%) and Korea (3.83%). Without such outliers their sample has a jump occrence average average of 1.17%.

[INSERT TABLE 2]

Next, we test for jumps in the variances of the foreign exchange series. Variance dynamics play a role in determining covariance dynamics of a price series pair. Further, a determining factor of $Vech$, from Equation (2), is the underlying stochastic volatility of the series. This becomes especially important in the latter simulation exercise. We provide variance jump dynamics results in Table 3. We find variance jumps occur in only 88 sample days (24.25%), with a mean prevalence of 1.43%. Therefore, while being more prevalent, fewer total days exhibit variance jumps than their price or covariance counterparts. Positive and negative jumps have the largest deviation with positive jumps occurring in 0.09% more observations and also being larger in magnitude.

[INSERT TABLE 3]

We lastly employ our test for predicting jumps in the covariance matrix. A separate test is conducted over each of the 15 Forex currency pairs, for a total of 105 tests. We begin by calculating the diffusive intraday covariance matrix, $Vech(\Sigma)$, according to Equation (13). The joint test for the occurrence of a jump in the $Vech(\Sigma)$, for a k-block is given by Equation (14). Average results from our covariance tests, along with price and variance test averages for reference, are provided in Table 4.[14] The mean covariance jump prevalence is 1.53% across 90 sample days. This suggestis that covariance jumps occur approximately 62.8% more often than price jumps and 7% more than variance jumps. Positive jumps occur slightly more often (0.02%) than negative counterparts. Covariance jumps are also the smallest in magnitude.

We conducted additional tests using alternative frequency specifications of $k = 15 minutes$, $30 minutes$, and $1 hour$. For brevity, results for each of these are not provided here. Overall results indicate that covariance jumps become more prevalent and larger in magnitude as time-intervals increase.[15] They also occur in a larger proportion of days. Such results are similar to those found by Lee and Mykland (2008) and Lee and Wang (2019), and further reiterate the value of including volatility in model specifications.[16] Our results additionally suggest that covariance jumps often occur contemporaneously to jumps in price, while not being a precondition for their existence.

[INSERT TABLE 4]

4.2. Monte Carlo Simulation

In this section, we examine the performance of our covariance matrix jump estimator using simulated data. We conduct the simulation following a two stage process. In stage one, we simulate a covariance series according to an affine jump process. Jumps in the intraday covariance pattern are parameterized according Merton (1976) where the the jump component follows a poisson process described by:

$$N_t = \sum_{n \geq 1} 1_{t \geq \tau_n} \tag{15}$$

[14] Our jump test results for each of the diffusive covariance asset pair can be found in Appendix 1. We do not present the full table here for brevity, instead opting for reporting average statistics.

[15] Results at each frequency are available upon request.

[16] Eraker et al. (2003) highlights that models without volatility jump components are mispecified.

where the jump intensity parameter λ, is the expected number of jumps per time unit. Combining the previous equations, a series under the Merton Jump-Diffusion model is characterized by:

$$P_t = P_0 e^{\mu - \frac{\sigma^2}{2} t + \sigma W_t + \sum_{i=t}^{N^t} Q_t} \tag{16}$$

where μ is the diffusion drift, σ is diffusion volatility, W_t a standard Brownian motion, and $\{\Sigma_{i=1}^{N_t} Q_i\}_{t\geq 0}$ a compound Poisson process. We set the probability of jump occurrence (intensity), λ equal to 0.1 (10%), initial covariance to 0.5, and volatility at 0.3.

In the second stage, a similar jump diffusion process is used to simulate covariance paths for each of the price pairs. This results in two price series following a doubly stochastic jump process as described in Equation (1) and Equation (2). We show the paths for the underlying simulated covariance processes in the first colums of Figure 1. 5000 paths were replicated under each covariance calibration with varying jump sizes of 0.1, 0.5, 1, and 2 times the volatility level. We further restrict values to be greater than zero, to ensure the diffusive covariance matrix remains positive semi definite. The resultant covariance paths were then used in determining two corresponding price series. Price series were calibrated with an initial price of 100 and identically sized jumps occurring at $\lambda = 10\%$. Each of our price series simulations are shown in the second and third columns of Figure 1, respectively. Final simulated series correspond to one month (31 days) of trading.[17]

[INSERT FIGURE 1]

We then examine whether our tests sufficiently detect jumps in the covariance ($Vech(\Sigma)$) matrix in the simulated series at a 5% level of significance. Multiple frequency samplings of 5 minutes, 30 minutes, 1 hour and 12 hours and each of the aforementioned jump magnitudes

[17] One month of trading days is equal to 31 days. Simulation was conducted at frequencies of k=5 minutes, 30 minutes, 1 hour, and 12 hours. This results in 8928, 2976, 744, and 62 monthly observations per path.

are explored by varying the mean jump size by a scalar multiple of the volatility level, σ. We focus upon two estimated metrics. The first, *Global Failure to Detect* (GFTD) is the proability of our test not detecting a jump. We calculate GFTD as the difference between the total number of detected jumps and the number of occurring jumps imposed by our calibrated model. Therefore, the total detection rate over the entire sample is $1 - GFTD$. The second metric is *Global Spurious Detection* (GSD). GSD is the probability of the test to experience a Type I error of detecting a jump that did not occur. Consistently we find the GSD of our test to be near zero.[18]

Table 5 presents our results. Overall, we fail to detect jumps at quite low rates under various high-frequency settings. At a baseline level, meaning a 5 minute frequency and sample corresponding volatility, our test detects 99.67% of jumps. Even at the 12 hour frequency, 91.8% of jumps are detected. Successful detection rates increase at higher data frequencies, while test accuracy decreases significantly with jump size. Taken together,our test performs quite similar to asymptotic results at high observational frequency and under large jump sizes. Correspondingly, as frequency and size are reduced the tests' precision decreases. We assessed the sensitivity of our test to alternative parameter specifications and initializing values across each observational frequency fingind our test statistics to be quite robust, with negligible changes to global and spurious detection rates occurring. We take these results as confirmation that our covariance jump test performs well under a wide variety of potential scenarios.

[INSERT TABLE 5]

[18]For an in depth mathematical discussion of such tests see Lee and Mykland (2008), as these are primarily extensions from their work.

5. Conclusion

We have proposed a new method for identifying intraday jumps in the diffusion covariance matrix of high-frequency financial assets. Previously literature shows that inefficient estimation of model components leads to errors in pricing models. Our method has two distinct advantages. First, it avoids model misspecification errors corresponding to asset jump-diffusions processes. Second, it provides standard normally distributed and χ^2 test statistics.

Test performance was evaluated using tick-by-tick exchange rate and simulated data. We find covariance jumps to occur at a rate of 1.53%, while variance and price jumps occur at rates of 1.42% and 1.06% respectively. A subsequent Monte-Carlo simuation study shows our estimator correctly identifies jumps in diffusive covariance matrices with virtually zero spurious detection. Under the commonly used 5 minute frequency with sample equal jump size variance, covariance jumps are properly identified over 99% of the time. However, it should be noted that the test performs best under high frequency and sufficiently large jump magnitude settings. Empirical examinations provided evidence of the strong association between covariance and price jumps, with covariance jumps occurring at a higher rate.

Our results inform a wide variety of prior literature examining intraday jumps in asset prices, as well as those estimating or forecasting second moment processes. While we have not explored the many individual implications, our test provides an accessible framework for the parameterization of such models. One key limitation of our study is the required precondition of the diffusive matrix being non-independent and resultant covariances being greater than zero. Future research may look into relaxing these assumptions or utilizing alternative parameterizations such as the matrix-logarithm of Bauer and Vorkink (2006) and fractional logit procedures.

References

Andersen, T. G., L. Benzoni, and J. Lund (2002). An empirical investigation of continuous-time equity return models. *The Journal of Finance 57*(3), 1239–1284.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys (2001). The distribution of realized exchange rate volatility. *Journal of the American statistical association 96*(453), 42–55.

Andersen, T. G., T. Bollerslev, and D. Dobrev (2007). No-arbitrage semi-martingale restrictions for continuous-time volatility models subject to leverage effects, jumps and iid noise: Theory and testable distributional implications. *Journal of Econometrics 138*(1), 125–180.

Bali, T. G. (2008). The intertemporal relation between expected returns and risk. *Journal of Financial Economics 87*(1), 101–131.

Bali, T. G. and R. F. Engle (2010). The intertemporal capital asset pricing model with dynamic conditional correlations. *Journal of Monetary Economics 57*(4), 377–390.

Barndorff-Nielsen, O. E. and N. Shephard (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of financial econometrics 2*(1), 1–37.

Barndorff-Nielsen, O. E. and N. Shephard (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of financial Econometrics 4*(1), 1–30.

Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *The Review of Financial Studies 9*(1), 69–107.

Bates, D. S. (2000). Post-'87 crash fears in the s&p 500 futures option market. *Journal of econometrics 94*(1-2), 181–238.

Bauer, G. H. and K. Vorkink (2006). Multivariate realized stock market volatility. *Available at SSRN 938151*.

Bekaert, G. and G. Wu (2000). Asymmetric volatility and risk in equity markets. *The review of financial studies 13*(1), 1–42.

Bibinger, M., M. Jirak, and M. Vetter (2017). Nonparametric change-point analysis of volatility. *The Annals of Statistics 45*(4), 1542–1578.

Black, F. (1976). Studies of stock market volatility changes. *1976 Proceedings of the American statistical association bisiness and economic statistics section*.

Branger, N., C. Schlag, and E. Schneider (2008). Optimal portfolios when volatility can jump. *Journal of Banking & Finance 32*(6), 1087–1097.

Callot, L. A., A. B. Kock, and M. C. Medeiros (2017). Modeling and forecasting large realized covariance matrices and portfolio choice. *Journal of Applied Econometrics 32*(1), 140–158.

Campbell, J. Y. and L. Hentschel (1992). No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of financial Economics 31*(3), 281–318.

Caporin, M., E. Rossi, and P. S. d. Magistris (2015). Volatility jumps and their economic determinants. *Journal of Financial Econometrics 14*(1), 29–80.

Chernov, M. and E. Ghysels (2000). A study towards a unified approach to the joint estimation of objective and risk neutral measures for the purpose of options valuation. *Journal of financial economics 56*(3), 407–458.

Duan, J.-C., P. Ritchken, and Z. Sun (2006). Approximating garch-jump models, jump-diffusion processes, and option pricing. *Mathematical Finance: An International Journal of Mathematics, Statistics and Financial Economics 16*(1), 21–52.

Duffie, D., J. Pan, and K. Singleton (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica 68*(6), 1343–1376.

Duffie, D. and K. J. Singleton (1993). Simulated moments estimation of markov models of asset prices. *Econometrica: Journal of the Econometric Society*, 929–952.

Epps, T. W. (1979). Comovements in stock prices in the very short run. *Journal of the American Statistical Association 74*(366a), 291–298.

Eraker, B. (2004). Do stock prices and volatility jump? reconciling evidence from spot and option prices. *The Journal of Finance 59*(3), 1367–1403.

Eraker, B., M. Johannes, and N. Polson (2003). The impact of jumps in volatility and returns. *The Journal of Finance 58*(3), 1269–1300.

Fiszeder, P. (2018). Low and high prices can improve covariance forecasts: The evidence based on currency rates. *Journal of Forecasting 37*(6), 641–649.

Fiszeder, P., M. Fałdziński, and P. Molnár (2019). Range-based dcc models for covariance and value-at-risk forecasting. *Journal of Empirical Finance 54*, 58–76.

Gallant, A. R. and G. Tauchen (1996). Which moments to match? *Econometric theory 12*(4), 657–681.

Gourieroux, C., A. Monfort, and E. Renault (1993). Indirect inference. *Journal of applied econometrics 8*(S1), S85–S118.

Guo, H. and R. F. Whitelaw (2006). Uncovering the risk–return relation in the stock market. *The Journal of Finance 61*(3), 1433–1463.

Harvey, C. R. (1991). The world price of covariance risk. *The Journal of Finance 46*(1), 111–157.

Hayashi, T. and N. Yoshida (2005). On covariance estimation of non-synchronously observed diffusion processes. *Bernoulli 11*(2), 359–379.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The review of financial studies 6*(2), 327–343.

Jacod, J. and V. Todorov (2010). Do price and volatility jump together? *The Annals of Applied Probability 20*(4), 1425–1469.

Jacquier, E., N. G. Polson, and P. E. Rossi (2002). Bayesian analysis of stochastic volatility models. *Journal of Business & Economic Statistics 20*(1), 69–87.

Johannes, M. (2004). The statistical and economic role of jumps in continuous-time interest rate models. *The Journal of Finance 59*(1), 227–260.

Lee, S. S. and P. A. Mykland (2008). Jumps in financial markets: A new nonparametric test and jump dynamics. *The Review of Financial Studies 21*(6), 2535–2563.

Lee, S. S. and M. Wang (2019). The impact of jumps on carry trade returns. *Journal of Financial Economics 131*(2), 433–455.

Lin, B.-H., M.-W. Hung, J.-Y. Wang, and P.-D. Wu (2013). A lattice model for option pricing under garch-jump processes. *Review of Derivatives Research 16*(3), 295–329.

Liu, J., F. A. Longstaff, and J. Pan (2003). Dynamic asset allocation with event risk. *The Journal of Finance 58*(1), 231–259.

Liu, J. and J. Pan (2003). Dynamic derivative strategies. *Journal of Financial Economics 69*(3), 401–430.

Melino, A. and S. M. Turnbull (1995). Misspecification and the pricing and hedging of long-term foreign currency options. *Journal of International Money and Finance 14*(3), 373–393.

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of financial economics 3*(1-2), 125–144.

Moskowitz, T. J. (2003). An analysis of covariance risk and pricing anomalies. *The Review of Financial Studies 16*(2), 417–457.

Pan, J. (2002). The jump-risk premia implicit in options: Evidence from an integrated time-series study. *Journal of financial economics 63*(1), 3–50.

Rossi, A. G. and A. Timmermann (2015). Modeling covariance risk in merton's icapm. *The Review of Financial Studies 28*(5), 1428–1461.

Todorov, V. (2015). Jump activity estimation for pure-jump semimartingales via self-normalized statistics. *The Annals of Statistics 43*(4), 1831–1864.

Todorov, V. and G. Tauchen (2011). Volatility jumps. *Journal of Business & Economic Statistics 29*(3), 356–371.

Todorov, V., G. Tauchen, and I. Grynkiv (2014). Volatility activity: Specification and estimation. *Journal of Econometrics 178*, 180–193.

Wu, L. (2003). Jumps and dynamic asset allocation. *Review of Quantitative Finance and Accounting 20*(3), 207–243.

Zhang, L. (2006). Efficient estimation of stochastic volatility using noisy observations: A multi-scale approach. *Bernoulli 12*(6), 1019–1043.

Zhang, L., P. A. Mykland, and Y. Aït-Sahalia (2005). A tale of two time scales: Determining integrated volatility with noisy high-frequency data. *Journal of the American Statistical Association 100*(472), 1394–1411.

Figures

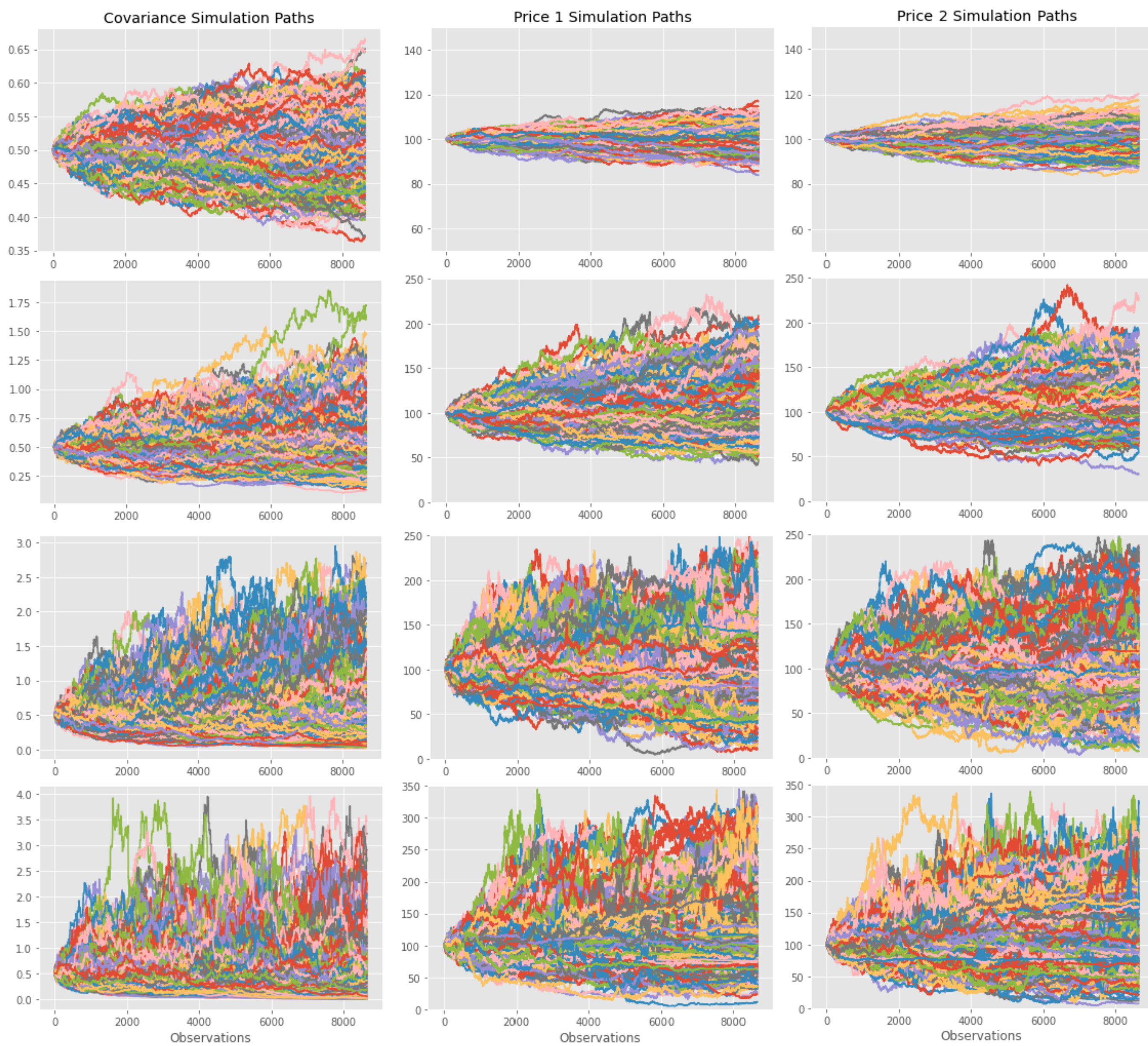

Figure 1. Note: Probabilities representative of simulated series. $\lambda = 0.1$. Each row refers to the volatility level, $X\sigma$, used for the simulation. X is a scalar multiple (from top to bottom row), where $X = [0.1, 0.5, 1, 2]$. Initial covolotility is set at 0.5 and price set at 100. Observations are simulated over a 31 day trading month for $k = 5minutes$. Each graph includes 5000 path simulations.

Tables

Table 1. Exchange Rate Return Statistics

Country	Code	Mean	St.Dev	Min	Max	Skewness	Kurtosis
Australia	AUD	0.025	0.476	-1.334	1.876	0.255	1.112
Canada	CAD	-0.021	0.438	-1.511	1.493	-0.369	1.639
Switzerland	CHF	-0.015	0.394	-1.288	2.021	0.158	2.513
Czech Rep.	CZK	-0.062	0.457	-1.404	2.014	-0.017	1.525
Denmark	DKK	-0.043	0.417	-1.402	1.560	-0.234	0.898
Euro	EUR	0.044	0.418	-1.583	1.399	0.232	0.900
Hungary	HUF	-0.042	0.495	-1.622	1.780	-0.191	1.169
Japan	JPY	-0.013	0.474	-1.741	1.466	-0.080	0.854
Mexico	MXN	-0.017	0.713	-2.489	1.839	-0.167	0.826
New Zealand	NZD	0.005	0.514	-1.923	1.625	-0.046	0.781
Russia	RUB	-0.018	0.616	-1.832	2.260	0.228	0.912
Sweden	SEK	-0.034	0.491	-1.480	2.063	-0.047	0.833
Singapore	SGD	-0.025	0.243	-0.821	0.851	-0.062	0.932
Turkey	TRY	0.022	0.760	-2.887	2.742	0.184	1.820
South Africa	ZAR	-0.033	0.874	-3.038	2.769	-0.106	1.235

Note: This table reports the first four moments of daily returns for the full sample of currencies. Returns a presented in percentage terms.

Table 2. Price Jump Statistics

Currency	All Jumps			Positive Jumps				Negative Jumps				Jump Days	
	#Tests	J#	J %	%	Q1	Q2	Q3	%	Q1	Q2	Q3	%	#
AUD	104148	1012	0.97%	0.50%	2.41E-04	4.65E-04	1.15E-03	0.47%	2.28E-04	4.70E-04	1.06E-03	44.20%	160
CAD	104148	1094	1.05%	0.54%	1.57E-04	3.43E-04	9.93E-04	0.51%	1.82E-04	3.78E-04	1.00E-03	34.53%	125
CHF	104148	911	0.87%	0.45%	2.55E-04	4.37E-04	1.05E-03	0.43%	2.75E-04	5.43E-04	4.82E-03	36.46%	132
CZK	104148	1107	1.06%	0.54%	2.30E-04	5.46E-04	1.35E-03	0.52%	2.67E-04	6.11E-04	1.49E-03	33.15%	120
DKK	104148	1078	1.04%	0.52%	1.90E-04	4.00E-04	1.10E-03	0.51%	2.08E-04	4.09E-04	1.24E-03	34.53%	125
EUR	104148	1059	1.02%	0.51%	2.04E-04	4.21E-04	1.24E-03	0.51%	1.94E-04	3.85E-04	1.11E-03	34.81%	126
HUF	104148	1046	1.00%	0.51%	2.67E-04	6.37E-04	1.36E-03	0.49%	2.67E-04	5.85E-04	1.42E-03	31.49%	114
JPY	104148	1047	1.01%	0.51%	2.15E-04	4.54E-04	9.75E-04	0.50%	2.62E-04	5.14E-04	1.30E-03	36.19%	131
MXN	104141	1213	1.16%	0.60%	2.76E-04	5.66E-04	1.11E-03	0.56%	3.28E-04	6.22E-04	6.22E-04	39.78%	144
NZD	104148	965	0.93%	0.46%	3.08E-04	5.50E-04	1.28E-03	0.46%	2.77E-04	5.33E-04	1.23E-03	45.03%	163
RUB	104147	1449	1.39%	0.72%	5.74E-04	1.50E-03	2.40E-03	0.67%	6.32E-04	1.44E-03	2.47E-03	24.03%	87
SEK	104148	997	0.96%	0.48%	2.50E-04	5.47E-04	1.36E-03	0.47%	2.65E-04	5.62E-04	1.47E-03	37.29%	135
SGD	104148	1051	1.01%	0.51%	1.05E-04	2.35E-04	4.54E-04	0.50%	1.15E-04	2.58E-04	5.82E-04	47.24%	171
TRY	104148	1267	1.22%	0.61%	3.32E-04	8.16E-04	1.62E-03	0.61%	3.00E-04	6.52E-04	1.33E-03	34.53%	125
ZAR	104148	1251	1.20%	0.61%	4.03E-04	1.03E-03	1.98E-03	0.60%	4.48E-04	1.04E-03	2.02E-03	38.95%	141
Average	104147	1103	1.06%	0.54%	2.67E-04	5.97E-04	1.30E-03	0.52%	2.83E-04	6.00E-04	1.54E-03	36.81%	133

Note: This table presents price jump prevalence and corresponding statistics for each currency series. 'Jump Days' refers to trading days which experience a price jump. k = 5 minutes. Sample period (full): January 1 to December 31, 2017.

Table 3. Variance Jump Statistics

Currency	All Jumps			Positive Jumps				Negative Jumps				Jump Days	
	#Tests	J#	J %	%	Q1	Q2	Q3	%	Q1	Q2	Q3	%	#
AUD	104147	1576	1.51%	0.82%	1.32E-08	5.45E-08	1.26E-07	0.69%	1.98E-09	6.23E-09	1.57E-08	24.58%	89
CAD	104147	1622	1.56%	0.84%	3.12E-09	1.41E-08	4.36E-08	0.72%	2.60E-08	1.21E-07	2.94E-07	21.27%	78
CHF	104147	1442	1.38%	0.75%	3.93E-08	9.59E-08	1.86E-07	0.64%	4.48E-09	1.02E-08	2.38E-08	23.75%	86
CZK	104147	1650	1.58%	0.85%	1.61E-05	6.36E-05	1.53E-04	0.73%	1.99E-06	6.04E-06	1.81E-05	23.20%	84
DKK	104147	1635	0.82%	0.75%	1.30E-07	3.40E-07	1.07E-06	0.75%	1.30E-07	3.40E-07	1.07E-06	22.37%	81
EUR	104147	1553	1.49%	0.82%	2.45E-08	1.03E-07	2.23E-07	0.67%	4.32E-09	1.12E-08	3.25E-08	23.20%	84
HUF	104146	1505	1.45%	0.76%	3.64E-03	1.13E-02	2.27E-02	0.69%	3.39E-04	8.96E-04	2.33E-03	22.37%	81
JPY	104147	1599	1.54%	0.82%	4.52E-04	1.31E-03	3.09E-03	0.72%	5.05E-05	1.31E-04	3.62E-04	25.41%	92
MXN	104140	1605	1.54%	0.82%	1.57E-05	7.06E-05	1.47E-04	0.73%	7.65E-07	4.95E-06	1.89E-05	24.30%	88
NZD	104147	1369	1.31%	0.69%	2.53E-08	6.77E-08	1.54E-07	0.63%	2.09E-09	6.22E-09	1.76E-08	29.55%	107
RUB	104145	1393	1.34%	0.69%	9.76E-04	2.46E-03	5.74E-03	0.64%	8.21E-06	1.12E-05	1.93E-04	27.90%	101
SEK	104146	1521	1.46%	0.77%	3.09E-06	9.90E-06	2.12E-05	0.69%	2.98E-07	9.32E-07	2.70E-06	22.92%	83
SGD	104147	1552	1.49%	0.81%	1.98E-08	6.23E-08	1.41E-07	0.68%	8.20E-10	5.18E-09	1.59E-08	24.86%	90
TRY	104147	1515	1.45%	0.79%	9.39E-07	3.02E-06	7.06E-06	0.67%	1.86E-08	1.84E-07	8.10E-07	24.03%	87
ZAR	104147	1521	1.46%	0.78%	2.96E-05	8.95E-05	1.74E-04	0.68%	6.87E-07	5.42E-06	1.81E-05	24.03%	87
Average	104146	1537	1.43%	0.78%	3.42E-04	1.02E-03	2.14E-03	0.69%	2.68E-05	7.04E-05	1.96E-04	24.25%	88

Note: This table presents variance jump prevalence and corresponding statistics for each currency series. ‘Jump Days’ refers to trading days which experience a price jump. k = 5 minutes. Sample period (full): January 1 to December 31, 2017.

Table 4. Average Jump Statistics

	All Jumps			Positive Jumps				Negative Jumps				Jump Days	
Series	#Tests	J#	J %	%	Q1	Q2	Q3	%	Q1	Q2	Q3	%	#
Price	104147	1103	1.06%	0.54%	2.67E-04	5.97E-04	1.30E-03	0.52%	2.83E-04	6.00E-04	1.54E-03	36.81%	133
Variance	104146	1537	1.43%	0.78%	3.42E-04	1.02E-03	2.14E-03	0.69%	2.68E-05	7.04E-05	1.96E-04	24.25%	88
Covariance	104146	1594	1.53%	0.78%	2.06E-06	1.45E-05	5.75E-05	0.75%	1.49E-06	9.99E-06	4.18E-05	24.76%	90

Note: This table presents average jump prevalence and corresponding statistics across all currency series. 'Jump Days' refers to trading days which experience a price jump. k = 5 minutes. Sample period (full): January 1 to December 31, 2017.

Table 5. Probability of Misclassification

	1 - GFTD				GSD
Block Size	0.1σ	0.5σ	1σ	2σ	All
k = 5 minutes	0.8728	0.9959	0.9967	0.9987	0.0000
	(0.1082)	(0.0346)	(0.0346)	(0.0316)	(0.0000)
k = 30 minutes	0.7927	0.9571	0.9931	0.9948	0.0000
	(0.1417)	(0.0660)	(0.0489)	(0.0433)	(0.0000)
k = 1 hours	0.6501	0.9081	0.9888	0.9982	0.0000
	(0.1464)	(0.1071)	(0.0418)	(0.0353)	(0.0000)
k = 12 hours	0.4383	0.7403	0.9180	0.9852	0.0000
	(0.0931)	(0.1464)	(0.0827)	(0.0402)	(0.0000)

Note: This table presents covariance jump test performance statistics. Probabilities representative of simulated price series. $\lambda = 0.1$. $X\sigma$ refers to the volatility level, where X is a scalar multiple. k denotes the frequency of observations. Initial covolotility set at 0.5. $GFTD$ is the global probability of the test failing to detect a jump in covariance. GSD is the global probability of the test spuriously detecting a jump in covariance.Observations are simulated over a 31 day trading month for each k. Values reflect the mean across 5000 simulations.

Appendix 1: Covariance Jump Statistics (1)

	All Jumps			Positive Jumps				Negative Jumps				Jump Days	
Series	#Tests	J#	J %	%	Q1	Q2	Q3	%	Q1	Q2	Q3	%	#
AUD_CAD	104147	1668	1.60%	0.83%	8.58E-10	3.36E-09	1.23E-08	0.77%	8.00E-09	3.96E-08	1.03E-07	23.20%	84
AUD_CHF	104147	1513	1.45%	0.75%	9.75E-10	3.63E-09	1.25E-08	0.70%	4.85E-09	3.06E-08	8.18E-08	25.69%	93
AUD_CZK	104147	1521	1.46%	0.78%	2.49E-08	1.00E-07	3.79E-07	0.75%	5.79E-08	3.77E-07	1.51E-06	24.03%	87
AUD_DKK	104147	1587	1.52%	0.78%	5.03E-09	1.78E-08	6.07E-08	0.74%	2.72E-08	1.68E-07	5.01E-07	24.31%	88
AUD_EUR	104147	1565	1.50%	0.77%	6.17E-09	3.36E-08	8.99E-08	0.73%	8.95E-10	3.23E-09	1.22E-08	25.97%	94
AUD_HUF	104146	1450	1.39%	0.71%	5.28E-07	2.34E-06	8.10E-06	0.69%	5.04E-07	2.31E-06	9.03E-06	26.52%	96
AUD_JPY	104147	1585	1.52%	0.79%	1.18E-07	4.40E-07	1.57E-06	0.74%	4.03E-07	2.34E-06	8.87E-06	26.24%	95
AUD_MXN	104140	1685	1.62%	0.82%	1.33E-08	1.30E-07	6.00E-07	0.80%	1.39E-08	1.13E-07	5.13E-07	27.07%	98
AUD_NZD	104147	1588	1.52%	0.79%	7.28E-09	3.86E-08	9.08E-08	0.73%	8.03E-10	2.70E-09	8.63E-09	27.62%	100
AUD_RUB	104145	1534	1.47%	0.74%	2.08E-30	3.41E-07	2.14E-06	0.73%	2.42E-10	5.50E-07	3.24E-06	26.24%	95
AUD_SEK	104146	1540	1.48%	0.74%	9.33E-09	3.64E-08	1.35E-07	0.74%	3.27E-08	1.78E-07	6.08E-07	24.86%	90
AUD_SGD	104147	1591	1.53%	0.77%	5.92E-10	2.50E-09	7.20E-09	0.76%	2.27E-09	1.67E-08	5.74E-08	26.80%	97
AUD_TRY	104147	1639	1.57%	0.80%	2.88E-09	1.59E-08	5.60E-08	0.78%	9.10E-09	1.04E-07	3.41E-07	22.38%	81
AUD_ZAR	104147	1592	1.53%	0.77%	1.97E-08	9.45E-08	3.68E-07	0.76%	4.91E-08	4.10E-07	1.36E-06	25.41%	92
CAD_CHF	104147	1493	1.43%	0.74%	6.13E-09	3.67E-08	1.06E-07	0.69%	1.28E-09	5.54E-09	1.93E-08	24.59%	89
CAD_CZK	104147	1628	1.56%	0.79%	9.07E-08	5.66E-07	1.87E-06	0.78%	3.20E-08	1.53E-07	6.47E-07	22.65%	82
CAD_DKK	104147	1652	1.59%	0.80%	3.90E-08	2.20E-07	6.73E-07	0.79%	5.25E-09	2.50E-08	9.44E-08	23.20%	84
CAD_EUR	104147	1566	1.50%	0.76%	1.03E-09	5.12E-09	1.96E-08	0.74%	7.56E-09	4.09E-08	1.19E-07	24.03%	87
CAD_HUF	104146	1572	1.51%	0.77%	8.97E-07	3.90E-06	1.53E-05	0.74%	6.84E-07	3.43E-06	1.34E-05	24.59%	89
CAD_JPY	104147	1630	1.57%	0.81%	4.79E-07	2.89E-06	1.11E-05	0.76%	1.48E-07	6.29E-07	2.46E-06	24.59%	89
CAD_MXN	104140	1818	1.75%	0.88%	1.67E-08	1.62E-07	8.79E-07	0.87%	2.05E-08	1.72E-07	8.09E-07	25.41%	92
CAD_NZD	104147	1554	1.49%	0.77%	9.15E-10	3.39E-09	1.16E-08	0.72%	8.35E-09	3.98E-08	1.11E-07	24.86%	90
CAD_RUB	104145	1674	1.61%	0.81%	6.40E-09	7.01E-07	3.66E-06	0.80%	2.55E-09	6.06E-07	3.49E-06	24.59%	89
CAD_SEK	104146	1635	1.57%	0.79%	3.40E-08	2.25E-07	7.19E-07	0.78%	1.39E-08	6.12E-08	2.11E-07	22.93%	83
CAD_SGD	104147	1605	1.54%	0.78%	2.51E-09	2.33E-08	7.21E-08	0.76%	6.22E-10	3.21E-09	1.10E-08	25.97%	94
CAD_TRY	104147	1694	1.63%	0.82%	1.24E-08	1.15E-07	4.42E-07	0.81%	4.06E-09	2.56E-08	9.54E-08	23.20%	84
CAD_ZAR	104147	1605	1.54%	0.78%	6.98E-08	5.71E-07	1.87E-06	0.76%	2.95E-08	1.61E-07	6.16E-07	24.86%	90
CHF_CZK	104147	1511	1.45%	0.73%	1.55E-07	8.87E-07	2.17E-06	0.72%	3.20E-08	1.10E-07	3.94E-07	22.93%	83
CHF_DKK	104147	1612	1.55%	0.79%	8.52E-08	3.77E-07	8.22E-07	0.76%	5.73E-09	2.10E-08	7.21E-08	23.20%	84
CHF_EUR	104147	1611	1.55%	0.80%	1.58E-09	5.10E-09	1.43E-08	0.74%	1.71E-08	6.71E-08	1.50E-07	22.93%	83
CHF_HUF	104146	1456	1.40%	0.71%	1.10E-06	4.03E-06	1.29E-05	0.69%	9.22E-07	4.15E-06	1.33E-05	24.03%	87
CHF_JPY	104147	1588	1.52%	0.79%	1.00E-06	5.00E-06	1.40E-05	0.74%	1.43E-07	5.35E-07	1.96E-06	25.41%	92
CHF_MXN	104140	1653	1.59%	0.80%	2.37E-08	1.72E-07	6.88E-07	0.79%	2.63E-08	1.73E-07	8.35E-07	26.24%	95
CHF_NZD	104147	1423	1.37%	0.69%	1.02E-09	3.86E-09	1.21E-08	0.68%	7.88E-09	3.82E-08	8.86E-08	25.41%	92

Appendix 1: Covariance Jump Statistics Continued (2)

	All Jumps			Positive Jumps				Negative Jumps				Jump Days	
Series	#Tests	J#	J %	%	Q1	Q2	Q3	%	Q1	Q2	Q3	%	#
CHF_RUB	104145	1625	1.56%	0.79%	2.03E-09	5.67E-07	2.99E-06	0.77%	2.68E-10	5.17E-07	2.71E-06	25.41%	92
CHF_SEK	104146	1477	1.42%	0.71%	5.69E-08	2.94E-07	8.77E-07	0.70%	1.52E-08	6.14E-08	1.82E-07	23.48%	85
CHF_SGD	104147	1544	1.48%	0.74%	4.74E-09	2.59E-08	7.04E-08	0.74%	6.99E-10	3.00E-09	9.64E-09	24.86%	90
CHF_TRY	104147	1529	1.47%	0.75%	1.23E-08	1.20E-07	3.56E-07	0.72%	6.13E-09	2.65E-08	1.02E-07	24.03%	87
CHF_ZAR	104147	1478	1.42%	0.71%	7.49E-08	5.01E-07	1.73E-06	0.71%	2.87E-08	1.75E-07	6.34E-07	24.59%	89
CZK_DKK	104147	1617	1.55%	0.80%	1.43E-06	7.99E-06	1.95E-05	0.76%	9.51E-08	4.09E-07	1.52E-06	24.03%	87
CZK_EUR	104147	1575	1.51%	0.77%	2.72E-08	1.00E-07	3.27E-07	0.74%	1.82E-07	1.10E-06	2.59E-06	23.76%	86
CZK_HUF	104146	1494	1.43%	0.72%	2.97E-05	1.41E-04	5.25E-04	0.71%	1.90E-05	7.60E-05	2.55E-04	25.41%	92
CZK_JPY	104147	1568	1.51%	0.76%	9.00E-06	5.10E-05	1.96E-04	0.74%	4.00E-06	1.70E-05	6.70E-05	26.52%	96
CZK_MXN	104140	1789	1.72%	0.87%	5.54E-07	4.71E-06	1.82E-05	0.85%	4.18E-07	4.08E-06	2.25E-05	23.48%	85
CZK_NZD	104147	1519	1.46%	0.74%	3.33E-08	1.14E-07	3.80E-07	0.72%	9.15E-08	4.83E-07	1.59E-06	26.52%	96
CZK_RUB	104145	1553	1.49%	0.75%	1.25E-07	1.88E-05	9.32E-05	0.74%	8.56E-08	1.64E-05	8.14E-05	24.59%	89
CZK_SEK	104146	1612	1.55%	0.80%	4.00E-06	1.60E-05	3.70E-05	0.75%	3.44E-07	1.25E-06	4.29E-06	23.20%	84
CZK_SGD	104147	1727	1.66%	0.85%	1.29E-07	7.79E-07	2.15E-06	0.81%	1.94E-08	7.18E-08	2.58E-07	22.65%	82
CZK_TRY	104147	1617	1.55%	0.79%	2.43E-07	2.49E-06	8.79E-06	0.76%	1.15E-07	6.96E-07	2.52E-06	22.10%	80
CZK_ZAR	104147	1681	1.61%	0.82%	2.00E-06	1.80E-05	5.50E-05	0.79%	6.70E-07	3.35E-06	1.10E-05	23.76%	86
DKK_EUR	104147	1613	1.55%	0.80%	4.99E-09	2.33E-08	8.53E-08	0.75%	1.30E-07	4.84E-07	1.15E-06	22.65%	82
DKK_HUF	104146	1512	1.45%	0.73%	6.00E-06	2.40E-05	9.40E-05	0.72%	5.00E-06	2.30E-05	8.00E-05	24.31%	88
DKK_JPY	104147	1589	1.53%	0.78%	5.00E-06	2.40E-05	7.70E-05	0.75%	7.84E-07	2.89E-06	1.07E-05	24.31%	88
DKK_MXN	104140	1753	1.68%	0.86%	1.42E-07	9.88E-07	4.24E-06	0.82%	1.15E-07	1.00E-06	5.00E-06	25.69%	93
DKK_NZD	104147	1469	1.41%	0.74%	5.90E-09	2.06E-08	6.36E-08	0.67%	3.79E-08	2.00E-07	5.66E-07	25.69%	93
DKK_RUB	104145	1628	1.56%	0.79%	6.22E-08	4.22E-06	2.22E-05	0.77%	6.39E-08	4.26E-06	1.94E-05	24.03%	87
DKK_SEK	104146	1598	1.53%	0.78%	6.82E-07	3.23E-06	7.37E-06	0.75%	4.27E-08	1.95E-07	6.65E-07	22.65%	82
DKK_SGD	104147	1723	1.65%	0.84%	3.15E-08	1.98E-07	5.43E-07	0.81%	4.01E-09	1.53E-08	4.58E-08	24.03%	87
DKK_TRY	104147	1679	1.61%	0.81%	1.06E-07	9.11E-07	2.71E-06	0.80%	2.65E-08	1.37E-07	5.58E-07	21.55%	78
DKK_ZAR	104147	1582	1.52%	0.78%	4.19E-07	4.20E-06	1.24E-05	0.74%	1.82E-07	8.69E-07	2.86E-06	23.20%	84
EUR_HUF	104146	1568	1.51%	0.76%	9.51E-07	4.13E-06	1.38E-05	0.75%	9.16E-07	3.97E-06	1.39E-05	24.03%	87
EUR_JPY	104147	1633	1.57%	0.82%	1.49E-07	5.73E-07	2.24E-06	0.75%	9.30E-07	4.34E-06	1.48E-05	25.14%	91
EUR_MXN	104140	1791	1.72%	0.87%	1.86E-08	1.89E-07	8.21E-07	0.85%	1.85E-08	1.67E-07	7.04E-07	23.48%	85
EUR_NZD	104147	1512	1.45%	0.75%	6.46E-09	3.18E-08	9.46E-08	0.71%	1.08E-09	3.94E-09	1.45E-08	25.97%	94
EUR_RUB	104145	1637	1.57%	0.79%	1.24E-08	6.86E-07	3.20E-06	0.78%	4.97E-09	6.94E-07	3.66E-06	27.62%	100
EUR_SEK	104146	1520	1.46%	0.74%	1.04E-08	4.66E-08	1.56E-07	0.72%	8.03E-08	3.89E-07	1.01E-06	23.20%	84
EUR_SGD	104147	1636	1.57%	0.79%	6.24E-10	3.01E-09	9.11E-09	0.78%	3.59E-09	2.43E-08	7.91E-08	27.07%	98
EUR_TRY	104147	1664	1.60%	0.81%	4.38E-09	2.38E-08	9.15E-08	0.79%	9.03E-09	1.22E-07	4.30E-07	21.55%	78
EUR_ZAR	104147	1668	1.60%	0.80%	3.14E-08	1.38E-07	5.72E-07	0.80%	5.05E-08	5.43E-07	1.83E-06	24.86%	90
HUF_JPY	104146	1460	1.40%	0.70%	9.50E-05	4.08E-04	1.49E-03	0.70%	8.10E-05	3.84E-04	1.42E-03	25.14%	91
HUF_MXN	104140	1690	1.62%	0.81%	7.00E-06	5.90E-05	2.46E-04	0.81%	6.00E-06	5.40E-05	2.72E-04	25.97%	94

Appendix 1: Covariance Jump Statistics Continued (3)

	All Jumps			Positive Jumps				Negative Jumps				Jump Days	
Series	#Tests	J#	J %	%	Q1	Q2	Q3	%	Q1	Q2	Q3	%	#
HUF_NZD	104146	1359	1.30%	0.67%	5.08E-07	2.30E-06	8.78E-06	0.64%	6.71E-07	3.02E-06	1.10E-05	27.35%	99
HUF_RUB	104145	1494	1.43%	0.72%	2.19E-07	2.79E-04	1.21E-03	0.71%	1.71E-27	1.39E-04	7.60E-04	25.14%	91
HUF_SEK	104146	1492	1.43%	0.72%	1.20E-05	5.10E-05	1.85E-04	0.71%	6.00E-06	2.50E-05	9.50E-05	24.86%	90
HUF_SGD	104146	1480	1.42%	0.71%	5.58E-07	2.66E-06	1.11E-05	0.71%	4.62E-07	2.10E-06	7.73E-06	26.24%	95
HUF_TRY	104146	1555	1.49%	0.75%	2.00E-06	1.90E-05	6.80E-05	0.74%	2.00E-06	1.40E-05	5.70E-05	23.48%	85
HUF_ZAR	104146	1523	1.46%	0.73%	1.80E-05	1.26E-04	4.23E-04	0.73%	1.40E-05	7.10E-05	2.60E-04	24.59%	89
JPY_MXN	104140	1662	1.60%	0.80%	2.00E-06	1.40E-05	7.20E-05	0.79%	2.00E-06	1.90E-05	9.30E-05	27.90%	101
JPY_NZD	104147	1519	1.46%	0.75%	1.48E-07	4.75E-07	1.49E-06	0.71%	7.27E-07	3.94E-06	1.13E-05	27.35%	99
JPY_RUB	104145	1538	1.48%	0.74%	4.29E-28	6.89E-05	4.29E-04	0.74%	2.67E-09	7.54E-05	3.71E-04	25.69%	93
JPY_SEK	104146	1539	1.48%	0.75%	4.00E-06	2.30E-05	7.30E-05	0.73%	2.00E-06	7.00E-06	2.60E-05	24.86%	90
JPY_SGD	104147	1591	1.53%	0.77%	3.59E-07	2.59E-06	7.83E-06	0.76%	1.01E-07	4.01E-07	1.38E-06	27.35%	99
JPY_TRY	104147	1566	1.50%	0.75%	1.00E-06	1.00E-05	4.20E-05	0.75%	6.53E-07	3.05E-06	1.08E-05	23.20%	84
JPY_ZAR	104147	1537	1.48%	0.75%	5.00E-06	4.00E-05	1.64E-04	0.72%	3.00E-06	1.70E-05	6.60E-05	27.35%	99
MXN_NZD	104140	1602	1.54%	0.78%	1.15E-08	1.35E-07	5.69E-07	0.76%	1.19E-08	9.88E-08	4.35E-07	28.45%	103
MXN_RUB	104140	1551	1.49%	0.76%	4.91E-08	1.64E-05	9.61E-05	0.73%	1.66E-07	1.46E-05	7.72E-05	25.97%	94
MXN_SEK	104140	1685	1.62%	0.81%	1.74E-07	1.53E-06	7.01E-06	0.81%	2.08E-07	1.43E-06	7.37E-06	24.59%	89
MXN_SGD	104140	1690	1.62%	0.82%	1.19E-08	1.23E-07	5.25E-07	0.81%	1.36E-08	1.49E-07	6.36E-07	24.59%	89
MXN_TRY	104140	1837	1.76%	0.88%	4.65E-08	6.42E-07	3.78E-06	0.88%	4.28E-08	7.11E-07	3.73E-06	28.18%	102
MXN_ZAR	104140	1756	1.69%	0.86%	2.80E-07	4.49E-06	2.29E-05	0.83%	3.34E-07	4.87E-06	2.32E-05	25.41%	92
NZD_RUB	104145	1560	1.50%	0.75%	4.65E-09	4.74E-07	2.41E-06	0.75%	4.20E-09	5.59E-07	3.15E-06	24.59%	89
NZD_SEK	104146	1434	1.38%	0.69%	1.08E-08	4.46E-08	1.58E-07	0.69%	4.37E-08	2.33E-07	6.65E-07	26.24%	95
NZD_SGD	104147	1527	1.47%	0.75%	5.61E-10	2.25E-09	7.16E-09	0.72%	3.00E-09	2.17E-08	6.44E-08	27.35%	99
NZD_TRY	104147	1526	1.47%	0.73%	4.26E-09	1.91E-08	6.48E-08	0.73%	1.36E-08	1.18E-07	3.38E-07	24.03%	87
NZD_ZAR	104147	1509	1.45%	0.74%	2.33E-08	1.16E-07	4.09E-07	0.71%	5.76E-08	5.34E-07	1.58E-06	26.24%	95
RUB_SEK	104145	1588	1.52%	0.77%	2.48E-09	5.93E-06	3.47E-05	0.75%	6.12E-08	6.02E-06	2.92E-05	23.76%	86
RUB_SGD	104145	1580	1.52%	0.76%	2.96E-20	5.64E-07	3.22E-06	0.76%	5.16E-30	5.51E-07	2.70E-06	26.80%	97
RUB_TRY	104145	1751	1.68%	0.84%	1.10E-08	3.08E-06	1.56E-05	0.84%	1.10E-08	3.08E-06	1.56E-05	22.38%	81
RUB_ZAR	104145	1725	1.66%	0.83%	3.87E-08	1.52E-05	9.84E-05	0.82%	6.89E-08	1.38E-05	7.89E-05	23.20%	84
SEK_SGD	104146	1570	1.51%	0.77%	4.32E-08	2.73E-07	8.24E-07	0.74%	7.98E-09	3.51E-08	1.29E-07	25.97%	94
SEK_TRY	104146	1574	1.51%	0.76%	1.75E-07	1.07E-06	3.23E-06	0.75%	5.78E-08	3.16E-07	1.09E-06	22.65%	82
SEK_ZAR	104146	1598	1.53%	0.76%	1.00E-06	8.00E-06	2.30E-05	0.76%	3.33E-07	1.32E-06	4.13E-06	21.55%	78
SGD_TRY	104147	1639	1.57%	0.79%	1.29E-08	9.91E-08	2.97E-07	0.78%	3.49E-09	1.79E-08	6.72E-08	23.20%	84
SGD_ZAR	104147	1674	1.61%	0.81%	5.79E-08	6.00E-07	1.98E-06	0.80%	1.92E-08	8.64E-08	2.96E-07	25.14%	91
TRY_ZAR	104147	1776	1.71%	0.87%	3.15E-07	4.32E-06	1.25E-05	0.84%	7.81E-08	5.19E-07	1.71E-06	22.10%	80
Average	104146	1594	1.53%	0.78%	2.06E-06	1.45E-05	5.75E-05	0.75%	1.49E-06	9.99E-06	4.18E-05	24.76%	90

Note: This table presents covariance jump prevalence and corresponding statistics across all currency pairs. ‘Jump Days’ refers to trading days which experience a price jump. k = 5 minutes. Sample period (full): January 1 to December 31, 2017.